

# **Neural Hamiltonian Deformation Fields** for Dynamic Scene Rendering







Hai-Long Qin<sup>1</sup>, Sixian Wang<sup>2</sup>, Guo Lu<sup>2</sup>, Jincheng Dai<sup>1</sup> Beijing University of Posts and Telecommunications (BUPT) <sup>2</sup>Shanghai Jiao Tong University (SJTU)





#### **Abstract**

Representing dynamic scenes with realistic motion remains challenging as existing methods often produce physically implausible deformations. We introduce NeHaD, a neural deformation field for dynamic Gaussian splatting governed by Hamiltonian mechanics. Our key innovation replaces MLP-based deformation with Hamiltonian neural networks that model Gaussians evolving along energy-conserving trajectories in phase space, ensuring natural dynamics. We introduce Boltzmann equilibrium decomposition for energy-aware static/dynamic separation, and employ symplectic integration with rigidity constraints to handle real-world dissipation. Additionally, we extend NeHaD to adaptive streaming through scaleaware mipmapping. Extensive experiments demonstrate that NeHaD achieves physically plausible dynamic rendering with good quality-efficiency trade-offs, representing the first application of Hamiltonian mechanics to neural Gaussian deformation.

#### **Motivation**

Just as human cognition relies on physical intuition, dynamic scene rendering should respect physical laws to achieve realistic results. Current 4D Gaussian splatting methods produce physically implausible motions<sup>[1]</sup>. Our key insight: Gaussian covariances naturally exist on symplectic manifolds, making Hamiltonian mechanics ideal for deformation modeling. Using Hamiltonian neural networks, we learn energy-conserving trajectories from data, embedding physical knowledge into Gaussian deformation fields for stable, coherent dynamics.

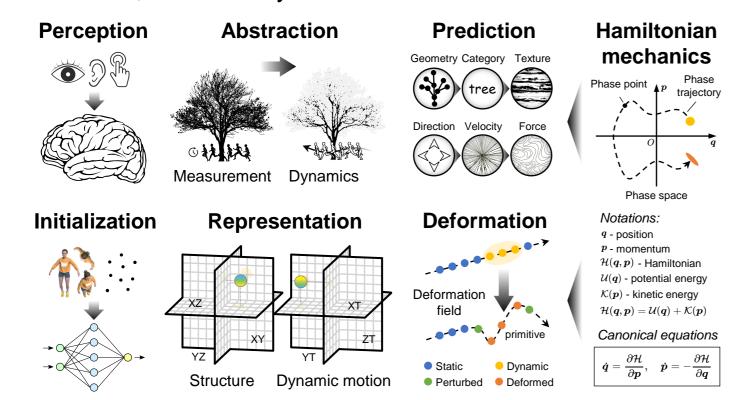


Figure 1. Both human cognition and scene rendering processes follow physical laws, with Hamiltonian mechanics offering mathematical frameworks aligned with physical intuition.

## **Methodology**

Neural Hamiltonian Deformation Fields. Replace MLPbased deformation with Hamiltonian neural networks (HNN) that learn scalar potentials generating conservative and solenoidal vector fields<sup>[2]</sup>

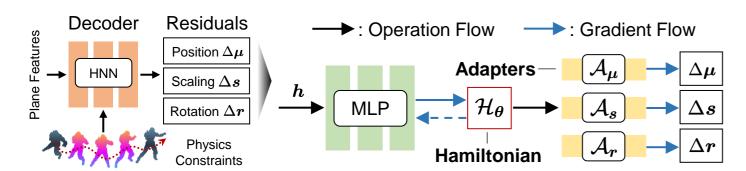


Figure 2. Through backpropagation of Hamiltonian gradients, the HNN optimizes vector fields and predicts Gaussian deformations via adapters.

**Boltzmann Equilibrium Decomposition.** (continued)

Adaptively separate static and dynamic Gaussians based on spatial-temporal energy deviation from equilibrium.

### **Physics-Informed Constraints.**

- Second-order symplectic integration: Position Verlet integration maintains conservation.
- Local rigidity regularization: ARAP-motivated constraints clamp rotation angles.

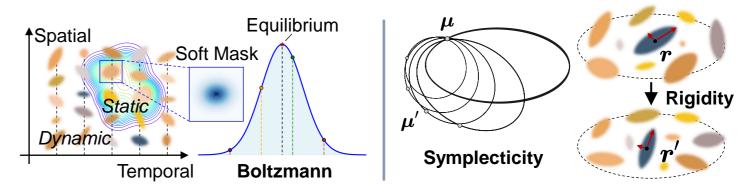


Figure 3. Left: decide which primitives should not be deformed with soft masks. Right: symplectic integration and rigidity regularization are used.

## **Experimental Results**

Performance. State-of-the-art rendering quality across all benchmarks with physically plausible dynamics and realtime capability. More results available in the full text.

D-NeRF [Pumarola et al. 2021] (monocular, synthetic, 800×800)						4DGS
Method	PSNR ↑	SSIM ↑	LPIPS ↓	Train Time ↓	FPS ↑	
D-NeRF	29.68	0.947	0.058	48 <i>hrs</i>	<1	
TiNeuVox	32.74	0.972	0.051	28 min	1.5	
K-Planes	31.52	0.967	0.047	52min	0.97	Donth consistency
HexPlane	31.04	0.97	0.04	11.5 <i>min</i>	2.5	Depth consistency
4DGS	35.34	0.985	0.021	<u>20 min</u>	<u>82</u>	4DGS
SC-GS	40.26	0.992	0.009	30 min	164	
Ours	40.91	0.995	0.008	24min	62	
HyperNeRF [Park et al. 2021b] (monocular, real-world, 536×960)						Towns and a share as
Method	PSNR↑	MS-SSIM↑	LPIPS ↓	Train Time ↓	FPS ↑	Temporal coherence
HyperNeRF	22.41	0.814	0.131	32hrs	<1	4DGS
TiNeuVox	24.20	0.836	0.128	30min	1	
4DGS	25.24	0.845	0.116	<u>34min</u>	32	
DeformGS	25.02	0.822	0.116	1.5 <i>hrs</i>	13	
SaRO-GS	25.38	0.850	0.110	1.2 <i>hrs</i>	<u>34</u>	
Grid4D	<u>25.50</u>	0.856	0.107	2.5 <i>hrs</i>	37	
Ours	25.69	0.858	0.104	45 min	25	Dynamical stability
DyNeRF [Li et al. 2022] (multi-view, real-world, 1352×1014)						4DG\$
Method	PSNR ↑	D-SSIM↓	LPIPS ↓	Train Time ↓	FPS ↑	
DyNeRF	29.58	0.020	0.083	1344 <i>hrs</i>	<1	
HexPlane	31.70	0.014	0.075	12hrs	0.2	
4DGS	31.17	0.016	0.049	<u>42 min</u>	30	
STG	32.05	0.014	0.044	10 <i>hrs</i>	<u>110</u>	
SaRO-GS	32.15	0.014	0.044	1.5 <i>hrs</i>	32	
C :0.4D	00.00	0.014	0.040	٠	105	

Table 1. Quantitative results.

0.013

32.23

32.35

Figure 4. Qualitative results.

Rendering accuracy

#### References.

Swift4D

Ours

[1] Wu G, et al. 4D Gaussian Splatting for Real-Time Dynamic Scene Rendering. CVPR 2024.

25min

50min

0.043

0.042

125

21

[2] Greydanus S, Dzamba M, Yosinski J. Hamiltonian Neural Networks. NeurlPS 2019.